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# $IIB$ supergravity and various aspects of light-cone formalism in AdS space-time [1]

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**Abstract**

Light-cone gauge manifestly supersymmetric formulation of type IIB 10-dimensional supergravity in $AdS_5 \times S^5$ background is discussed. The formulation is given entirely in terms of light-cone scalar superfield, allowing us to treat all component fields on an equal footing. Discrete energy spectrum of field propagating in AdS space is explained within the framework of light-cone approach. Light-cone gauge formulation of self-dual fields propagating in AdS space is developed. An conjectured interrelation of higher spin massless fields theory in AdS space-time and superstring theory in Minkowski space is discussed.

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**Light-cone gauge formulation of $IIB$ supergravity in $AdS_5 \times S^5$ background.** In recent years, due to the conjectured duality between the string theory and $\mathcal{N} = 4$, $4d$ SYM theory [1] (for review see [2]) there has been rapidly growing interest in strings propagating in AdS space. Inspired by this conjecture the Green-Schwarz formulation of strings propagating in $AdS_5 \times S^5$ was suggested in [3] (for further developments see [4]-[6]). Despite considerable efforts these strings have not yet been quantized[2]. As is well known, quantization of GS superstrings propagating in flat space is straightforward only in the light-cone gauge. Because the string theories are approximated at low energies by supergravity theories it seems reasonable that we should first study a light-cone gauge formulation of supergravity theories in AdS spacetime. Light-cone description of type $IIB$ supergravity in $AdS_5 \times S^5$ background can teach us a lot. For example it can be used to construct charges of global symmetries so that certain of them have the same form for both supergravity and superstring. Keeping in mind extremely important applications to string theory let us now restrict our discussion to $IIB$ supergravity in $AdS_5 \times S^5$.

Our method is conceptually very close to the one used in [11] (see also [12]) to find the light-cone form of $IIB$ supergravity in flat space and is based essentially on a light-cone gauge description of field dynamics developed recently in [13][3]. To discuss light-cone formulation we use the following parametrization of $AdS_5 \times S^5$ space

$$ds^2 = \frac{1}{z^2}(-dt^2 + dx_1^2 + dx_2^2 + dz^2 + dx_4^2) + \frac{1}{4} dy_{ij} dy^*_{ij} \,,$$

introduce light-cone variables $x^\pm \equiv (x^4 \pm x^0)/\sqrt{2}$, $x \equiv (x^1 + \mathrm{i}x^2)/\sqrt{2}$, $\bar{x} \equiv x^*$, where $x^0 \equiv t$ and treat $x^+$ as evolution parameter. Here and below we set the radii of both $AdS_5$ and $S^5$ equal to unity[4]. The coordinates $y_{ij}$ are related to the standard $so(6)$ cartesian coordinates $y^M$, $M = 1, \ldots, 6$, which satisfy the constraint $y^M y^M = 1$ through the formula $y_{ij} = \rho^M_{ij} y^M$, where $\rho^M_{ij}$ are the Clebsh Gordan coefficients of $su(4)$ algebra [16].

Our goal is to find a realization of $psu(2,2|4)$ superalgebra on the space of IIB supergravity fields propagating in $AdS_5 \times S^5$. To do that we use light-cone superspace formalism. First, we introduce light-cone superspace which by definition is based on position $AdS_5 \times S^5$ coordinates $x^\pm$, $x$, $\bar{x}$, $z$, $y^{ij}$ and Grassmann position coordinates $\theta^i$ and $\chi^i$ which transform in fundamental $\mathbf{4}$ irreps of $su(4)$ algebra. Second, on this superspace we introduce scalar superfield $\Phi(x^\pm, x, \bar{x}, z, y^{ij}, \theta^i, \chi^i)$. In the remainder of this paper we find it convenient to Fourier transform to momentum space for all coordinates except for $x^+$, $z$ and $S^5$ coordinates $y^{ij}$. This implies using $p^+$, $\bar{p}$, $p$, $\lambda_i$, $\tau_i$ instead of $x^-$, $x$, $\bar{x}$, $\theta^i$, $\chi^i$ respectively. The $\lambda_i$ and $\tau_i$ transform in $\bar{\mathbf{4}}$ irreps of $su(4)$. Thus we consider the superfield $\Phi(x^+, p^+, p, \bar{p}, z, y^{ij}, \lambda_i, \tau_i)$[5] which by definition satisfies the following reality constraint

$$\Phi(x^+, -p, z, y, \lambda, \tau) = p^{+4} \int d^4\lambda^\dagger d^4\tau^\dagger e^{(\lambda_i \lambda_i^\dagger + \tau_i \tau_i^\dagger)/p^+} (\Phi(x^+, p, z, y, \lambda, \tau))^\dagger \,.$$

In the light-cone formalism the $psu(2,2|4)$ superalgebra has the generators

$$P^+,\, P,\, \bar{P},\, J^{+x},\, J^{+\bar{x}},\, K^+,\, K,\, \bar{K},\, Q^{+i},\, Q^+_i,\, S^{+i},\, S^+_i,\, D,\, J^{+-},\, J^{x\bar{x}} \,, \qquad (1)$$

which we refer to as kinematical generators and

$$P^-,\, J^{-x}\,,\, J^{-\bar{x}},\, K^-\,,\, Q^{-i},\, Q^-_i\,,\, S^{-i},\, S^-_i \qquad (2)$$

[2]Some related interesting discussions are in [7]. Alternative approaches can be found in [8],[9]. Twistor-like formulations are discussed in [10].

[3]A discussion of $IIB$ supergravity at the level of gauge invariant equations of motion and actions can be found in [14] and [15] respectively.

[4]The $S^5$ coordinates $y^{ij}$ are subject to the constraints $y^{ik} y_{kj} = \delta^i_j$, $y_{ij} = \frac{1}{2}\epsilon_{ijkn} y^{kn}$, $y^*_{ij} = -y^{ij}$ where indices take the values $i, j, k, n = 1, 2, 3, 4$ The $\epsilon^{ijkn} = \pm 1$ is the Levi-Civita tensor of $su(4)$.

[5]An expansion of $\Phi$ in powers of Grassmann momenta $\lambda_i$ and $\tau_i$ can be found in [17].

which we refer to as dynamical generators. The kinematical generators have positive or zero $J^{+-}$ charges, while dynamical generators have negative $J^{+-}$ charges[6]. At a quadratic level both kinematical and dynamical generators have the following representation in terms of the physical light-cone superfield

$$\hat{G} = \int dp^+ d^2p dz dS^5 d^4\lambda d^4\tau \, p^+ \Phi(x^+, -p, z, y, -\lambda, -\tau) G \Phi(x^+, p, z, y, \lambda, \tau) \,,$$

where $G$ are the differential operators acting on $\Phi$. Thus we should find representation of $psu(2,2|4)$ in terms of differential operators acting on light-cone superfield $\Phi$. To simplify expressions let us write down the generators for $x^+ = 0$. The kinematical generators are then given by

$$P = p \,, \qquad \bar{P} = \bar{p} \,, \qquad P^+ = p^+ \,, \qquad J^{+x} = \partial_p p^+ \,, \qquad J^{+\bar{x}} = \bar{\partial}_p p^+ \,, \tag{3}$$

$$Q^{+i} = p^+\theta^i \,, \quad Q^+_i = \lambda_i \,, \quad S^+_i = \frac{1}{\sqrt{2}} z\tau_i - \lambda_i \partial_p \,, \quad S^{+i} = \frac{1}{\sqrt{2}} z p^+ \chi^i + p^+ \theta^i \bar{\partial}_p \,, \tag{4}$$

where $\partial_p \equiv \partial/\partial\bar{p}$, $\bar{\partial}_p \equiv \partial/\partial p$. Dynamical generator $P^-$ is given by

$$P^- = -\frac{p\bar{p}}{p^+} + \frac{\partial_z^2}{2p^+} - \frac{1}{2z^2 p^+} A \,, \qquad A \equiv X - \frac{1}{4} \,, \tag{5}$$

where

$$X \equiv l^{i2}_j + 4\tau l \chi + (\chi\tau - 2)^2, \quad l^{i2}_j \equiv l^i{}_j l^j{}_i \,, \quad \tau l \chi \equiv \tau_i l^i{}_j \chi^i \,, \qquad \chi\tau \equiv \chi^i \tau_i \,. \tag{6}$$

The $l^i{}_j$ is the orbital part of $su(4)$ angular momentum $J^i{}_j$. Explicit expressions for the remaining kinematical and dynamical generators may be found in [17][7]. Making use of the expression for $P^-$ (5) we can immediately write down the light-cone gauge action[8]

$$S_{l.c.} = \int dx^+ dp^+ d^2p dz dS^5 d^4\lambda d^4\tau \, p^+ \Phi(x^+, -p, z, y, -\lambda, -\tau)(\mathrm{i}\partial^- + P^-)\Phi(x^+, p, z, y, \lambda, \tau).$$

Following [13] we shall call the operator $A$ the $AdS$ mass operator. A few comments are in order.

(i) The operator $A$ is equal to zero only for massless representations realized as irreducible representations of conformal algebra [18],[13] which for the case of $AdS_5$ space is the $so(5,2)$ algebra. Because the operator $X$ (6) has eigenvalues equal to squared integers in the whole spectrum of compactification of $IIB$ supergravity on $AdS_5$ (see [17]) the operator $A$ is never equal to zero. This implies that the scalar fields [19] as well as all remaining fields of compactification of $IIB$ supergravity do not satisfy conformally invariant equations of motion.

(ii) The coordinate $\theta^i$ (or $\lambda_i$) constitutes odd part of light-cone superspace appropriate to superfield description of light-cone gauge $N=4$, $4d$ (or $N=1$, $10d$) SYM theory. The translation generators, Lorentz boosts and Poincaré supercharges given in (3),(4) take the same form as in SYM theory. From this we conclude that one can expect that superstring dynamics in $AdS_5 \times S^5$ could be presented as dynamics of free left and right movers appropriate to description of open superstrings supplemented by nonlinear dynamics of remaining degrees of freedom.

(iii) The generator $P^-$ involves terms up to fourth order in $\chi^i$ and $\tau_i$. The first thing to note is that terms of fourth order in $\chi^i$ and $\tau_i$ appear trough the number operator $\chi\tau$. The second point

[6]For $x^+ = 0$ the kinematical generators are quadratic in the physical field $\Phi$, while the dynamical generators receive corrections in interaction theory. Here we deal with free fields.

[7]For $\lambda_i$, $\tau_i$ and $\tau^i$, $\chi^i$ we adopt the following anticommutation rules $\{\theta^i, \lambda_j\} = \delta^i_j$, $\{\chi^i, \tau_j\} = \delta^i_j$.

[8]Since the action is invariant with respect to the symmetries generated by $psu(2,2|4)$ superalgebra, the formalism we discuss is sometimes referred to as an off shell light-cone formulation [12].

is that terms of fourth order can be excluded by introducing new superfield[9] $\Phi = z^{\chi\tau-2}\Phi^{new}$. On the space of $\Phi^{new}$ the hamiltonian $P^-$ takes the form

$$P^- = -\frac{p\bar{p}}{p^+} + \frac{\partial_z^2}{2p^+} + \frac{\chi\tau - 2}{zp^+}\partial_z - \frac{1}{2z^2p^+}(l^{i2}_{\ j} + 4\tau l\chi + \chi\tau - \frac{9}{4})$$

i.e. the terms of fourth order in $\chi^i$ and $\tau_i$ are absent. This suggests (but does not prove) light-cone gauge string action which does not involve higher than second order terms in anticommuting variables[10].

**Discrete energy spectrum of field in AdS space-time.** As is well known energy spectrum of field propagating in AdS space takes discrete values (see for review [21]). This phenomenon is not immediately visible in light-cone formulation. In this section we would like to explore how the discrete energy spectrum is obtained within the framework of the light-cone formulation.

The AdS translation generators $\hat{P}^a$ are expressible as[11] $\hat{P}^a = (1/2)P^a + K^a$. The $\hat{P}^0$ is the energy operator. In light-cone frame we have[12]

$$\hat{P}^0 = \frac{1}{2\sqrt{2}}(P^+ + 2K^+ - P^- - 2K^-)\,.$$

Representation of generators $P^a$ and $K^a$ on the space of physical fields has been found in [13]. For the case of totally symmetric fields this representation takes the form (for $x^+ = 0$)

$$\begin{aligned}
&P^+ = p^+\,, \qquad\qquad K^+ = \frac{1}{2}x_I^2 p^+\,, \qquad x_I^2 \equiv x^I x^I \\
&P^- = \frac{\partial_I^2}{2p^+} - \frac{1}{2z^2p^+}(-\frac{1}{2}M_{ij}^2 + \frac{(d-4)(d-6)}{4})\,, \qquad \partial_I^2 \equiv \partial^I\partial^I \\
&K^- = \frac{1}{2}x_I^2 P^- - \partial_{p^+}D + \frac{1}{2p^+}l^{IJ}M^{IJ} - \frac{x^I}{2zp^+}\{M^{zJ}, M^{JI}\}\,, \\
&D = -\partial_{p^+}p^+ + x^I\partial^I + \frac{d-2}{2}\,, \qquad M^{IJ} \equiv \alpha^I\bar{\alpha}^J - \alpha^J\bar{\alpha}^I\,.
\end{aligned} \tag{7}$$

where $l^{IJ} \equiv x^I\partial^J - x^J\partial^I$ and we adopt a convention: $\{a,b\} \equiv ab + ba$. The above generators act on physical field whose components are collected in Fock vector $|\phi\rangle$:

$$|\phi\rangle = \phi^{I_1\ldots I_s}\alpha^{I_1}\ldots\alpha^{I_s}|0\rangle\,, \qquad \bar{\alpha}^I|0\rangle = 0\,, \qquad [\bar{\alpha}^I, \alpha^J] = \delta^{IJ}\,, \tag{8}$$

where physical traceless tensor field $\phi^{I_1\ldots I_s}$ depends on $x^+, x^i, z, p^+$. Using these expressions we get the following representation for energy operator (for $x^+ = 0$)

$$2\sqrt{2}\hat{P}^0 = (1 + x_I^2)(p^+ - P^-) + 2\partial_{p^+}D - \frac{1}{p^+}l^{IJ}M^{IJ} + \frac{x^I}{zp^+}\{M^{zJ}, M^{JI}\}\,. \tag{9}$$

[9] We prefer to use superfield $\Phi$ instead of $\Phi^{new}$ because the $\Phi$ has conventional canonical dimension.

[10] Such action has been found recently in Ref.[20].

[11] In this section, in contrast to [13], we use hermitean $\hat{P}^a$, $P^a$, $K^a$ which are related with the anithermitean ones of [13] as follows $\hat{P}^a_{herm} = -\mathrm{i}\hat{P}^a_{antherm}$, $P^a_{herm} = -\mathrm{i}P^a_{antiherm}$, $K^a_{herm} = +\mathrm{i}K^a_{antiherm}$.

[12] We use parametrization of $AdS_d$ space in which $ds^2 = (-dx^{02} + dx_I^2 + dx_{d-1}^2)/z^2$. Light-cone coordinates in $\pm$ directions are defined as $x^\pm = (x^{d-1} \pm x^0)/\sqrt{2}$. From now on we adopt the following conventions: $I, J = 1, \ldots, d-2$; $i,j,k,l = 1,\ldots,d-3$. $\partial^I = \partial_I \equiv \partial/\partial x^I$, $\partial^+ = \partial_- \equiv \partial/\partial x^-$, $\partial_{p^+} \equiv \partial/\partial p^+$, $z \equiv x^{d-2}$. In momentum representation $\partial^+$ takes the form $\partial^+ = \mathrm{i}p^+$.

It is convenient to make the following transformation of wave function

$$\phi = U\tilde{\phi}\,, \qquad U \equiv (p^+)^{\frac{1}{2}(x^I\partial^I + \frac{d-5}{2})}$$

and use the formula

$$\partial_{p^+} D\phi = U\Big(-p^+\partial_{p^+}^2 - \frac{1}{2}\partial_{p^+} + \frac{1}{4p^+}(x_I^2\partial_J^2 - \frac{1}{2}l_{IJ}^2 + \frac{(d-3)(d-5)}{4})\Big)\tilde{\phi}. \tag{10}$$

Now to simplify our presentation let us restrict ourselves to the case of four dimensional ($d=4$) AdS space. For this case because the indices $i,j$ which label $d-3$ directions take one value $i,j=1$ the spin operator $M^{ij}$ is equal to zero. Now taking into account that $M_{IJ}^2 = 2M_{z1}^2$ and exploiting (10) in (9) we get the following representation of $\hat{P}^0$ in $\tilde{\phi}$

$$2\sqrt{2}\hat{P}^0 = H_1 + H_2\,, \tag{11}$$

where

$$H_1 \equiv -\frac{1}{2}\partial_I^2 + x_I^2\,, \tag{12}$$

$$H_2 \equiv -2p^+\partial_{p^+}^2 - \partial_{p^+} - \frac{1}{2p^+}\Big(2(\frac{1}{2}l^{IJ} + M^{IJ})^2 + \frac{1}{4}\Big) + p^+\,. \tag{13}$$

Because the hamiltonians $H_1$, $H_2$ commute with each other we can diagonalize them simultaneously. Let us find their eigenvalues. To this end we introduce complex coordinates $x$, $\bar{x}$ instead of $x^1, z$: $x \equiv (x^1 + \mathrm{i}z)/\sqrt{2}$, $\bar{x} \equiv x^*$. Next we decompose the wave function $|\tilde{\phi}\rangle$ as follows

$$|\tilde{\phi}\rangle = |\phi_{+s}\rangle + |\phi_{-s}\rangle\,,$$

where the $|\phi_{\pm s}\rangle$ are eigenvectors of $M^{x\bar{x}}$: $M^{x\bar{x}}|\phi_{\pm s}\rangle = \pm s|\phi_{\pm s}\rangle$. The $|\phi_{\pm s}\rangle$ themselves can be decomposed into eigenvectors of operator $l^{x\bar{x}}$

$$|\phi_{\pm s}\rangle = \sum_{m=-\infty}^{\infty} e^{\mathrm{i}m\varphi}|\phi_{\pm s,m}\rangle\,, \qquad l^{x\bar{x}}e^{\mathrm{i}m\varphi} = me^{\mathrm{i}m\varphi}\,, \qquad \varphi \equiv \arg(x^1 + \mathrm{i}z)\,.$$

On space of $|\phi_{\pm s,m}\rangle$ the hamiltonians $H_1$, $H_2$ take the form

$$H_1 = -\frac{1}{2}(\partial_r^2 + \frac{1}{r}\partial_r) + \frac{m^2}{2r^2} + r^2\,, \tag{14}$$

$$H_2 = -2p^+\partial_{p^+}^2 - \partial_{p^+} + \frac{1}{2p^+}\Big(\kappa^2 - \frac{1}{4}\Big) + p^+\,, \tag{15}$$

where $r$ is a radial variable $r \equiv |x^1 + \mathrm{i}z|$ and $\kappa \equiv m \pm 2s$. Because $H_1$ and $H_2$ commute with each other we can decompose the wave function as follows

$$|\phi_{\pm s,m}\rangle = \phi^{(1)}_{\pm s,m}(r)|\phi^{(2)}_{\pm s,m}(p^+)\rangle\,,$$

where $\phi^{(1)}_{\pm s,m}(r)$ and $|\phi^{(2)}_{\pm s,m}(p^+)\rangle$ are eigenvectors of $H_1$ and $H_2$ respectively. Introducing instead of $p^+$ a new variable $y$, by relation $y^2 = p^+$ and rescaling wave function

$$\phi^{(1)}_{\pm s,m}(r) = r^{-1/2}\tilde{\phi}^{(1)}_{\pm s,m}(r)\,,$$

we get the following hamiltonians

$$H_1 = -\frac{1}{2}\partial_r^2 + \frac{1}{2r^2}(m^2 - \frac{1}{4}) + \frac{\omega_0^2}{2}r^2\,, \qquad (16)$$

$$H_2 = -\frac{1}{2}\partial_y^2 + \frac{1}{2y^2}(\kappa^2 - \frac{1}{4}) + \frac{\omega_0^2}{2}y^2\,, \qquad (17)$$

where $\omega_0 \equiv \sqrt{2}$. The eigenvalues of the hamiltonians (16) and (17) responsible for square integrable eigenvectors are well known and are given by

$$E_1 = (2n_1 + |m| + 1)\omega_0\,, \qquad E_2 = (2n_1 + |m \pm 2s| + 1)\omega_0\,,$$

where $n_1, n_2 = 0, 1, \ldots$. Taking into account the relation (11) we get the following eigenvalues of $\hat{P}^0$

$$E = n_1 + n_2 + |\frac{m}{2} \pm s| + |\frac{m}{2}| + 1 \qquad (18)$$

A few comments are in order. (i) Our energy spectrum (as usual) is discrete. As compared to standard energy spectrum (see [21]) which depends on two integers our energy spectrum (18) depends on three integers $n_1$, $n_2$, $m$. Such the difference is well know from analysis of quantum mechanical energy spectrum of the standard three dimensional oscillator and is related to a choice of coordinates in which energy spectrum is evaluated. Normally, (see [21]) one uses global spherical coordinates while we use Poincaré coordinates. (ii) As usual for massless spin $s$ particle in $AdS_4$ our energy spectrum is bounded from below by $E_{min} = s + 1$. (iii) There is 2-fold degeneracy of lowest energy $E_{min}$ related to the two helicity states $|\phi_{\pm s}\rangle$. (iv) for each helicity state lowest energy has $(2s + 1)$-fold degeneracy, i.e. for $n_1 = n_2 = 0$ there exist $2s + 1$ values of $m$ for which $E$ given in (18) is equal to $E_{min}$. This degeneracy explains well known $so(3)$ symmetry of lowest energy state.

**Self-dual fields in AdS space-time.** In this section we would like to discuss self-dual fields in AdS space. In our knowledge they have not been discussed previously in literature. As in Minkowski space the strengths of AdS (anti) self-dual fields satisfy self-duality constraint

$$F_\pm^{\mu_1\ldots\mu_{d/2}} = \pm\frac{1}{(d/2)!}\frac{\epsilon^{\mu_1\ldots\mu_{d/2}\nu_1\ldots\nu_{d/2}}}{\sqrt{|g|}}F_{\pm\,\nu_1\ldots\nu_{d/2}} \qquad (19)$$

Because this constraint it is impossible to construct an appropriate Lorentz covariant action without introducing auxiliary fields[13]. The action for self-dual fields can be most easily understood within the framework of light-cone gauge formulation. For definiteness let us restrict ourselves to the case of six dimensional AdS space and to second rank antisymmetric tensor field. First of all we would like to discuss formulation of self-dual fields in Minkowski space which is most convenient for generalization to AdS space. In light-cone gauge physical degrees of freedom of self-dual field in Minkowski space are described by field $\phi^{IJ}$ which satisfies the self-duality constraint $\phi^{IJ} = (1/2)\epsilon^{IJKL}\phi^{KL}$. The field $\phi^{IJ}$, which is the $so(4)$ tensor, can be decomposed into $so(3)$ tensors $\phi^{ij}$ and $\phi^i$, $\phi^i \equiv \phi^{zi}$. The self-duality constraint tells us then that $\phi^{ij}$ is expressible in terms of $\phi^i$: $\phi^{ij} = \epsilon^{ijk}\phi^k$. Thus, if one wishes, in Minkowski space the self-dual field can be described by unconstrained field $\phi^i$ (or $\phi^{ij}$) [24][14]. Note that this form of description breaks

[13]Lorentz covariant formulations including auxiliary fields are discussed in [22],[23].

[14]It is worth mentioning that formulation suggested in [24] (and its generalization to curved space given in [25]) is extremely appropriate to description of self-dual fields in AdS space. The formulation given in [24] breaks Lorentz invariance $so(d-1,1)$ to $so(d-2,1)$ invariance and this does not fit with manifest Lorentz symmetry of Minkowski space metric. On the other hand in AdS space it is the $so(d-2,1)$ symmetry that is manifest symmetry of $AdS_d$ space metric considered in Poincaré coordinates (equivalently, the $so(d-2,1)$ is a manifest symmetry of AdS algebra considered in conformal algebra notation).

manifest $so(4)$ (which is $so(d-2)$ for $d=6$) invariance to manifest $so(3)$ (which is $so(d-3)$ for $d=6$) invariance. On the other hand light-cone gauge formalism in $AdS_d$ space respects only manifest $so(d-3)$ (see [13]). Therefore the description based on unconstrained field $\phi^i$ is most appropriate to be generalized to AdS space. Thus our aim is to find realization of AdS algebra generators on the space of unconstrained field $\phi^i$. The realization of AdS algebra on the space of physical fields found in ([13], formulas (4.1)-(4.8) and (4.17)-(4.19)) is formulated in terms of spin operator $M^{IJ}$ and operators $A$, $B$. The form of spin operator is fixed by representation of $so(4)$ algebra we interested in. To proceed we introduce creation operator $\alpha^i$ and construct Fock space vector

$$|\phi\rangle \equiv \phi^i\alpha^i|0\rangle\,, \qquad \bar\alpha^i|0\rangle = 0\,, \qquad [\bar\alpha^i,\alpha^j] = \delta^{ij}\,. \tag{20}$$

For this form of realization the spin operators take the form

$$M^{ij} = \alpha^i\bar\alpha^j - \alpha^j\bar\alpha^i\,, \qquad M^{zi} = \frac{1}{2}\epsilon^{ijk}M^{jk}\,. \tag{21}$$

Note that it is second relation in (21) that tells that we deal with self-dual representation of $so(4)$ algebra. As to above mentioned operators $A$ and $B$ they are fixed by defining equations found in ([13])

$$2\{M^{zi},A\} - [[M^{zi},A],A] = 0\,, \tag{22}$$

$$[M^{zi},[M^{zj},A]] + \{M^{iL},M^{Lj}\} = -2\delta^{ij}B\,. \tag{23}$$

where operator $A$ is invariant under $so(d-3)$ spin rotations, i.e. $[A,M^{ij}]=0$. From $[A,M^{ij}]=0$, the second relation in (21) and equations (22) we find the equation $\{A,M^{zi}\}=0$ which implies that $A=0$. By using (21) it is easy to get then the following relation

$$\{M^{iL},M^{Lj}\} = -2\delta^{ij}M^2_{zl}\,, \qquad M^2_{zl} \equiv M^{zl}M^{zl}\,.$$

From this relation and (23) we conclude that $B = M^2_{zi}$. It is easy to check that in the case under consideration the $M^2_{zi}$ is diagonalized: $M^2_{zi}|\phi\rangle = -2|\phi\rangle$. Due to (21) the $M^2_{zi}$ commutes with $M^{IJ}$ and therefore we can put $B=-2$. To summarize we have found

$$A = 0\,, \qquad B = -2\,. \tag{24}$$

Thus we found all entries of light-cone formulation. By substituting the expressions for spin operators (21) and operators $A$ and $B$ (24) in expressions (4.1)-(4.8) and (4.17)-(4.19) of Ref.[13] we find realization of AdS algebra generators on space of AdS self-dual field $|\phi\rangle$ (20). This provides complete description of self-dual field $|\phi\rangle$ (20) in $AdS_6$ space.

Generalization to the case of arbitrary spin $s$ self-dual fields in $AdS_6$ is straightforward. In this case we introduce

$$|\phi\rangle = \phi^{i_1\ldots i_s}\alpha^{i_1}\ldots\alpha^{i_s}|0\rangle\,,$$

where $\phi^{i_1\ldots i_s}$ is totally symmetric traceless $so(3)$ tensor field. The spin operators take the form given in (21) while for operators $A$ and $B$ we get

$$A = 0\,, \qquad B = -s(s+1)\,.$$

The light-cone gauge action for self-dual field in AdS takes then the form

$$S = \int d^dx\langle\partial^+\phi|(-\partial^- + P^-)|\phi\rangle\,, \qquad P^- = -\frac{\partial_I^2}{2\partial^+}\,,$$

which coincides with the one in Minkowski space. We think that this coincidence can be traced to the conformal invariance of self-dual fields [18]. Note that to describe anti self-dual fields one needs to replace the second relation in (21) by relation $M^{zi} = -(1/2)\epsilon^{ijk}M^{jk}$. All remaining formulas above given do not change their form.

**Superstring theory and AdS higher spin massless fields theory. Conjecture.** One of major motivations for our investigation of higher spin massless fields theory in the AdS space is to seek a possible relation between this theory and string theory. It seems rather attractive to conjecture that string theory can be interpreted as resulting from some kind of a spontaneous breakdown of higher spin symmetries. In [13] it has been conjectured that *superstrings could be considered as the ones living at the boundary of* 11*-dimensional AdS space while their unbroken (symmetric) phase is realized as the theory of higher spin massless fields living in this* $AdS_{11}$ *space.* AdS theories are symmetric with respect to isometry algebra of $AdS_d$ space which is $so(d-1,2)$. This algebra is not realized however as symmetry algebra of string S-matrix . This implies that $so(d-1,2)$ symmetry should be (spontaneously) broken. Here we would like to demonstrate that if one restricts attention to totally symmetric fields and make some mild assumptions about the (spontaneously) broken form of AdS theory hamiltonian $P^-$ then some interesting and non-trivial test of our conjecture can be carried out. To this end we consider the $P^-$ for AdS totally symmetric fields (8). As was demonstrated in [13] one has

$$P^- = -\frac{\partial_I^2}{2\partial^+} + \frac{1}{2z^2\partial^+}A\,, \qquad A = -\frac{1}{2}M_{ij}^2 + \frac{(d-4)(d-6)}{4}\,. \tag{25}$$

where $M^{ij}$ is given in (7). This $P^-$ can be rewritten as

$$P^- = \frac{-\partial_i^2 + M^2}{2\partial^+}\,, \qquad M^2 \equiv -\partial_z^2 + \frac{1}{z^2}A\,. \tag{26}$$

The operator $M^2$ in (26) can be interpreted as mass operator for a field propagating in $(d-1)$ dimensional Minkowski spacetime while the $P^-$ (26) can be considered as hamiltonian of this field. The operator $M^2$ given in (26) has continuous spectrum. This implies that AdS theories which are not supplemented by (spontaneous) symmetry breaking lead to boundary theories which have continuous mass spectrum. In order to get theory with discrete mass spectrum one needs to break AdS symmetry. Now we have to make assumption about term which breaks AdS symmetry. Our suggestion is to consider the following ansatz for (spontaneously) broken $P^-_{s.b}$

$$P^-_{s.b} = P^- + \frac{\omega^2 z^2 - \bar\omega}{2\partial^+}\,, \qquad \omega \equiv \frac{1}{2\alpha'}\,, \qquad \bar\omega \equiv \frac{d-1}{2\alpha'}\,, \tag{27}$$

where $P^-$ is given in (25) and $\alpha'$ is the universal Regge slope parameter[15]. For this $P^-_{s.b}$ the mass operator takes the form

$$M^2_{s.b} = -\partial_z^2 + \frac{1}{z^2}A + \omega^2 z^2 - \bar\omega\,. \tag{28}$$

This $M^2_{s.b}$, in contrast to $M^2$ given in (26), has discrete spectrum. Let us evaluate this spectrum. To this end we decompose the field $|\phi\rangle$, which transforms in irreducible representation of $so(d-2)$ algebra, into irreducible representations of $so(d-3)$ subalgebra $|\phi_{s'}\rangle$

$$|\phi\rangle = \sum_{s'=0}^{s} \oplus |\phi_{s'}\rangle\,. \tag{29}$$

[15]Here we discuss open AdS and string theories. For the case of closed theories the $\omega$ and $\bar\omega$ in (27) should be multiplied by factor 2.

Because of relation $M^2_{ij}|\phi_{s'}\rangle = -2s'(s'+d-5)|\phi_{s'}\rangle$ the operator $M^2_{s.b}$ for $|\phi_{s'}\rangle$ takes the form

$$M^2_{s.b} = -\partial_z^2 + \frac{1}{z^2}(\nu^2 - \frac{1}{4}) + \omega^2 z^2 - \bar{\omega}\,, \qquad \nu \equiv s' + \frac{d-5}{2}\,. \tag{30}$$

The spectrum of this operator is well known and is given by

$$m^2_{s.b} = 2\omega(\nu + 2n + 1) - \bar{\omega}\,, \qquad n = 0, 1, \ldots\,. \tag{31}$$

From this it is seen that for leading term in decomposition (29), i.e. for $s' = s$, and for $n = 0$ the mass spectrum is given by

$$m^2_{s.b} = (s-1)/\alpha' \tag{32}$$

and this coincides exactly with the mass spectrum of massless and massive string states belonging to leading Regge trajectory. Note that overall normalization factor as well as additive constant in r.h.s of relation (32) are result of the form of conjectured term in (27) which breaks (spontaneously) AdS symmetry. What is extremely important is that the dependence on $s$ in (32) is essentially fixed by specific form of AdS mass operator $A$ given in (25).

By now due to [26] it is known that to construct self-consistent interaction of higher spin massless fields in $AdS_4$ it is necessary to introduce, among other things, a infinite chain of massless totally symmetric fields which consists of every spin just once [26]. In higher dimensions the totally symmetric fields should be supplemented by mixed symmetry fields. What is important however in higher dimensional AdS space it is also necessary to introduce the same infinite chain of massless totally symmetric fields, i.e. the one which consists of every spin just once. It is this chain of totally symmetric fields that can be found on leading Regge trajectory of string theory.

To summarize we have demonstrated that if (spontaneously) broken hamiltonian $P^-$ takes the form given in (27) then the *leading components of AdS massless totally symmetric arbitrary spin $s$ states $|\phi_s\rangle$ (29) become massive string states belonging to leading Regge trajectory.*

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